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The paradox of axions surviving primordial magnetic fields

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Abstract

In the presence of primordial magnetic fields the oscillating cosmic axion field drives an oscillating electric field. The ensuing dissipation of axions is found to be inversely proportional to the conductivity of the primordial plasma. This counterintuitive result is essentially equivalent to "Zeno's paradox" or the "watched-pot effect" of quantum mechanics. It implies that the standard predictions of the cosmic axion density remain unaltered even if primordial magnetic fields are strong.

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Besides neutralinos, axions are the only theoretically well-motivated particle candidate for the ubiquitous cold dark matter that appears to be required in the standard picture of cosmic structure formation. Primordial axions were created by the "misalignment mechanism" [1] as well as by the relaxation of the string network which formed at the Peccei-Quinn phase transition [2]. In units of the cosmic critical density the relic axion abundance is found to be $\Omega_a h^2 = \xi\ (10^{-5}\ \mathrm{eV}/m)^{1.175}$, where h is the present-day Hubble parameter in units of $100\ \mathrm{km\,s^{-1}Mpc^{-1}}$ and m the axion mass. The exact value of the numerical coefficient $\xi = \mathcal{O}(1)$ is the subject of some debate [2]. However, if axions are the dark matter in the galactic halos implied by astronomical observations, it appears safe to assume that their mass lies in the range $10^{-5}\ \mathrm{eV} \lesssim m \lesssim 10^{-3}\ \mathrm{eV}$. The current round of direct search experiments [3] for the first time has a realistic chance of detecting galactic axions at the lower end of this mass range.

Because of their nonthermal production, axions are essentially born as a Bose condensate, i.e. a classical, coherent field oscillation of the axion field. It is obviously important to understand if these oscillations are damped by dissipation effects which would thermalize, and thus reduce, the cosmic axion population. For example, it has been shown [4] that the thermalization by interactions with the cosmic plasma is inefficient for $m \lesssim 10^{-1}$ eV, corresponding to values of the Peccei-Quinn scale $f_a \gtrsim 10^8$ GeV. (The axion mass and the Peccei-Quinn scale are related by $m = 0.62\ \mathrm{eV}\ 10^7\ \mathrm{GeV}/f_a$.) A proposed "coherent" damping mechanism [5] does not seem to be effective in practice [6]. The possibility of resonant axion-photon conversion in a cosmological magnetic field of order 10^{-9} G has also been studied, with the conclusion that it yields no significant axion dissipation [7].

We presently study another dissipation mechanism which is expected if strong primordial magnetic fields exist. They may arise during the early cosmic phase transitions [8], and recently it has been shown that magnetic fields are indeed a stable feature of a second order (electroweak) phase transition [9]. Locally the field could be very large. It is limited only by primordial nucleosynthesis arguments, which imply that $B \lesssim 3\times 10^{10}$ G at $t \simeq 10^4$ s [10]. Because flux conservation implies that $B \sim R^{-2}$ (R is the cosmic scale factor), at earlier times the field could have been much stronger. On dimensional grounds, a typical scale for magnetic field fluctuations should be $B \sim T^2$ so that at the time of the electroweak phase transition local fields as high as 10^{24} G could obtain. Depending on how such a large, random magnetic field scales at large distances, it could be the seed field needed to explain the observed galactic magnetic fields [11]. Let us remark that such large magnetic fields will not facilitate resonant axion-photon conversion in the early universe because the magnetic field removes the possibility for degeneracy in the refractive indices of the axion and photon fields.

Magnetic fields would however couple to the cold axions and produce an oscillat-

ing electric field, an effect which is used in the cavity experiments which search for galactic axion dark matter [3]. In the early universe, such a field would give rise to a bulk velocity of the charge carriers (electrons, muons, and above the QCD phase transition temperature, quarks) and hence to a current. This induced current would rapidly dissipate by thermal collisions in the hot plasma. If the magnetic field is large this process might dissipate the cosmic axion energy density. Unexpectedly, however, we find this damping mechanism to be ineffective because the conductivity of the primordial plasma is *too large*. We believe that the somewhat paradoxical nature of the axion survival story makes it worthwhile to communicate these results.

In order to derive the equations of motion for axions coupled to the electromagnetic field with dissipation we start from the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu a \partial^\mu a - \tfrac{1}{2}m^2 a^2 + \tfrac{1}{2}\mathbf{E}^2 - \tfrac{1}{2}\mathbf{B}^2 + g_{a\gamma} a\, \mathbf{E}\cdot\mathbf{B}, \tag{1}$$

where a is the axion field, m its mass,

$$g_{a\gamma} \equiv \frac{\alpha}{2\pi f_a} \tag{2}$$

the axion-photon coupling constant, and f_a the Peccei-Quinn scale. Because of the Nambu-Goldstone nature of axions, the Lagrangian Eq. (1) is valid only for $a \ll f_a$.

The equations of motion for the coupled axion-photon system that follow from this Lagrangian have been derived by several authors [12]. In our case they simplify if we assume that the electromagnetic field is dominated by a large homogeneous primordial magnetic field $\mathbf{B}$ and an induced electric field $\mathbf{E}$ while we neglect a higher-order induced magnetic field as well as the thermal radiation fields. If we include the possibility of an electric current density $\mathbf{j}$ the coupled equations of motion are found to be

$$\begin{aligned} \dot{\mathbf{E}} &= -g_{a\gamma}\, \mathbf{B}\, \dot{a} - \mathbf{j}\,, \\ \ddot{a} + m^2 a &= g_{a\gamma}\, \mathbf{E}\cdot\mathbf{B}\,. \end{aligned} \tag{3}$$

Note that in the literature the terms proportional to $g_{a\gamma}$ are often presented with an erroneous relative sign.

The only conceivable macroscopic current density $\mathbf{j}$ is the one induced by the electric field $\mathbf{E}$. Assuming a linear response of the medium we may use Ohm's law $\mathbf{j} = \sigma\mathbf{E}$ where σ is the conductivity of the primordial plasma. From Eq. (3) it is then evident that $\mathbf{B}$ and $\mathbf{E}$ are parallel so that we may use $E = |\mathbf{E}|$ and $B = |\mathbf{B}|$ instead. This leads to our final equations of motion

$$\begin{aligned} \dot{E} &= -\beta\dot{a} - \sigma E, \\ \ddot{a} + m^2 a &= \beta E, \end{aligned} \tag{4}$$

where $\beta \equiv g_{a\gamma} B$.

The overall behavior of this system is perhaps easiest to understand if one uses the vector potential A as a dynamical electric field variable by virtue of $E = -\dot{A}$. After a Fourier transformation the equation of motion is

$$\begin{pmatrix} \omega^2 + i\sigma\omega & -i\beta\omega \\ -i\beta\omega & -\omega^2 + m^2 \end{pmatrix} \begin{pmatrix} A \\ a \end{pmatrix} = 0, \tag{5}$$

revealing that we have to do with two coupled harmonic oscillators of which one is damped, *i.e.* with an axion-photon mixing phenomenon [13]. This equation has solutions only if the determinant of the matrix vanishes, giving us the dispersion relations.

One obvious solution is a static mode $\omega_1 = 0$, corresponding to a constant A and a vanishing a, i.e. to no electric or axion field at all. In the absence of dissipation ($\sigma = 0$), there is a second static mode $\omega_2 = 0$, even in the presence of mixing ($\beta \neq 0$). In the presence of dissipation this mode is purely damped, *i.e.* ω_2 is always purely imaginary. For large dissipation ($\sigma \gg m$ or β) one finds $\omega_2 = -i\sigma$. The two remaining modes are $\omega_{3,4} = \pm m$ if there is no magnetic field ($\beta = 0$), *i.e.* they correspond to the axion field. It obtains an electric field admixture for $\beta \neq 0$.

In order to understand better the behavior of these mixed modes it is useful to consider a number of approximations. The natural oscillation frequency is the axion mass m. The "mixing energy" β is proportional to $1/f_a$ and thus to m; numerically $\beta = 3.65 \times 10^{-21}\,(B/\mathrm{G})\,m$. We only consider magnetic field strengths small enough that always $\beta \ll m$ so that we are in the "weak mixing limit". Any deviation from $\omega_{3,4} = \pm m$ will then be of order β^2. The shift of the real part of $\omega_{3,4}$ is $\pm\frac{1}{2}\beta^2/m$ for $\sigma = 0$, and less for $\sigma > 0$. The imaginary (damping) part of these frequencies is

$$\mathrm{Im}(\omega_{3,4}) = \tfrac{1}{2}\beta^2 \times \begin{cases} 2\sigma/m^2 & \text{for } \sigma \ll m, \\ 1/\sigma & \text{for } \sigma \gg m. \end{cases} \tag{6}$$

Therefore, if $\sigma \ll m$ the axion field is damped more strongly for an increasing conductivity as naively expected. In the "strong damping limit" $\sigma \gg m$, on the other hand, the actual damping rate of the axion modes decreases with increasing σ.

Actually the conductivity of the primordial plasma is huge. In the regime $m_e \ll T \ll T_{\mathrm{QCD}}$ one finds for an isotropic relativistic electron gas [14]

$$\sigma = \frac{\omega_{\mathrm{plas}}^2}{4\pi\sigma_{\mathrm{coll}} n_e} \simeq \frac{T}{3\pi\alpha}, \tag{7}$$

where ω_{plas} is the plasma frequency and σ_{coll} the collision cross section. This result is valid for fields smaller than the critical field $B_c = m_e^2/e = 4.41 \times 10^{13}$ G, above which the electrons cannot be treated as free, and the conductivity Eq. (7) should be multiplied by a factor B/B_c.

In the nonrelativistic regime, relevant for the recombination time, one should use the conductivity of a nonrelativistic isotropic hydrogen plasma, given by

$$\sigma \simeq \frac{(2T)^{3/2}}{5\pi^{3/2}\alpha m_e^{1/2}}. \tag{8}$$

Either way, the conductivity is very large compared with the axion mass so that we are always in the "strong damping limit."

Because the axion field amplitude is dissipated away at the rate $\Gamma = \beta^2/(2\sigma)$, the rate for the dissipation of axion number density is that of the squared amplitude, $\Gamma = \beta^2/\sigma$. This enables us to check whether this damping mechanism is effective or not in an expanding universe in the following way.

The damping of axions would be effective only if the dissipation rate for the axion number density exceeds the Hubble expansion rate, *i.e.* if damping would occur on a time scale fast compared with the cosmic expansion time scale. Assuming magnetic flux conservation β scales as $1/R^2$ and hence essentially as T^2, while σ scales as T in the relativistic epoch so that $\Gamma \propto T^3$. The expansion rate is roughly $T^2/m_{\rm Pl}$ (Planck mass $m_{\rm Pl}$) so that axions must be dissipated early if they are dissipated at all. One crudely estimates $\Gamma/H \approx (\beta/m)^2 m^2 m_{\rm Pl}/T^3$. Even if β/m were near unity at the QCD epoch, the factor $m^2 m_{\rm Pl}/T_{\rm QCD}^3$ is sufficiently below unity for the axion masses under consideration. Therefore, axions are not significantly damped by this mechanism.

This conclusion remains valid if one takes the expansion of the universe explicitly into account, and also if one considers the nonrelativistic epoch as we shall now demonstrate. In order to study the explicit solutions of Eq. (3) it is useful to take $\Theta = a/f_a$ as a dynamical variable describing the axion field. With the initial conditions $\Theta(t_0) = 1$ and $E(t_0) = 0$, the approximate solution for $\beta \ll m \ll \sigma$ is

$$\Theta(t) \simeq \exp(-\frac{\beta^2(t-t_0)}{2\sigma})\cos[m(t-t_0)]. \tag{9}$$

However, this still neglects the expansion of the universe which we shall now include.

Assuming a flat Robertson-Walker metric with $R(t)$ the dimensionless scale factor of the universe, we define

$$F_{0i} = -RE_i; \quad F_{ij} = \epsilon_{ijk} B_k R^2 \ . \tag{10}$$

Including dissipation as before, one may then derive the following equations of motion:

$$\begin{aligned} \dot{E} &= -\frac{gB_0\dot{\Theta}}{R^2} - 2HE - \sigma E, \\ \ddot{\Theta} + 3H\dot{\Theta} + m^2\Theta &= \frac{gEB_0}{f_a^2 R^2}. \end{aligned} \tag{11}$$

Here $g = \alpha/(2\pi) = g_{a\gamma} f_a$ and $H = \dot{R}/R$ is the Hubble parameter, and we have made use of flux conservation which entails $R^2 B = \text{const} = B_0$, so that in a comoving volume the ratio of the magnetic energy density to radiation density remains fixed.

In this case one cannot perform a simple mode analysis as in the nonexpanding case. Instead, eliminating E from Eq. (11), one obtains a third order differential equation for Θ:

$$\dddot{\Theta} + (7H+\sigma)\ddot{\Theta} + (m^2 + \beta^2 R^{-4} + 12H^2 + 3\dot{H} + 3H\sigma)\dot{\Theta} + (4Hm^2 + \sigma m^2)\Theta = 0 \quad (12)$$

where we have used $\beta \equiv g_{a\gamma} B_0 = g B_0 / f_a$. Assuming a radiation dominated universe, we may find an approximate solution to Eq. (12) by writing $\Theta(t) = C(t)\Theta_0(t)$, where $\Theta_0(t)$ is the solution in the absence of expansion, and $C(t)$ is a slowly varying function. Note that $\sigma \gg m \gg H$ except exactly at the QCD phase transition, above which the axion mass is actually smaller than H. The mass starts to turn on very rapidly, however, so that the hierarchy of scales is valid when $T \lesssim T_{\rm QCD}$. Writing $\sigma = \sigma_0 (t_0/t)^{1/2}$, to lowest order in β and $H = 1/(2t)$ we then obtain

$$\Theta(t) \simeq \left(\frac{t_0}{t}\right)^{3/4} \exp\left[\frac{\beta^2 t_0}{\sigma_0}\left((\frac{t_0}{t})^{1/2} - 1\right)\right] \cos[m(t-t_0)] \ . \quad (13)$$

One readily observes that damping cannot compete with the expansion of the universe, which redshifts the energy of the background magnetic field, and at $t \to \infty$ damping tends to an asymptotic value. Thus the decrease in the axion field density due to magnetic field catalysed dissipation in the early universe is completely negligible. For example, for a critical field $B \simeq 10^{13}$ G one finds, substituting Eq. (7) into Eq. (13), that dissipation reduces Θ by a factor $(1-5)\times 10^{-22}$. This result will remain qualitatively unchanged even if we allow for magnetic fields greater than B_c.

In the nonrelativistic regime the conductivity is given by Eq. (8) and $H = 2/(3t)$, whence Eq. (13) is replaced by

$$\Theta(t) \simeq \left(\frac{t_0}{t}\right) \exp\left[\frac{3\beta^2 t_0}{4\sigma_0}\left((\frac{t_0}{t})^{2/3} - 1\right)\right] \cos[m(t-t_0)] \ . \quad (14)$$

Therefore, damping effects are again negligible.

The standard predictions of the cosmic axion density remain unaltered even if there exist strong primordial magnetic fields. The reason is that the electric field never has a chance to grow large because the ohmic dissipation is so effective. The pump turning the axion energy into electric energy produces a mere trickle.

This rather counterintuitive result is closely related to "Zeno's paradox" or the "watched-pot effect" [15]. If two quantum states mix, the transition rate between them is suppressed if one of them is repeatedly measured, causing the system to remain "frozen" in the original state. In our case axions would oscillate into photons because the two fields are coupled by the external magnetic field. The dissipation of the electric field energy can be viewed on the quantum level as a photon absorption and thus as a "measurement" of the system to be in the electromagnetic state. If it is measured too frequently, it stays frozen in the initial axion state: the transition rate is suppressed inversely proportional to the rate of measurement.

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